Purpose: In this problem set, you will improve your understanding of logarithmic functions by studying their algebraic properties.

1. Let's get warmed up with some computations. Your notes from Friday will come in handy.

   (a) Evaluate $\log_5(125)$

   (b) Evaluate $\log_3\left(\frac{1}{27}\right)$

   (c) Evaluate $\log_7(1)$

   (d) Evaluate $\log_8\left(\frac{1}{4}\right)$. (Hint: put 8 and 1/4 in base 2.)

Here's a summary of logarithmic functions for your reference:

- General Logarithm Function:

$$y = \log_b(x), \text{ for } b > 0.$$

- Common Logarithm Function:

$$y = \log_{10}(x) = \log(x).$$

- Natural Logarithm Function:

$$y = \log_e(x) = \ln(x).$$

Now, we need to build a list of the algebraic properties of logarithms.

2. Let $b > 0$.

   (a) $b^0 = 1$, so $\log_b(\_\_\_) = \_\_\_$.

   (b) $b^1 = b$, so $\log_b(\_\_\_) = \_\_\_$.

   (c) $b^x b^y = b^{x+y}$ so $\log_b(b^x b^y) =$ _____. Reformulated: _____

   (d) $\dfrac{b^x}{b^y} = b^{x-y}$ so $\log_b\left(\dfrac{b^x}{b^y}\right) =$ _____. Reformulated: _____

   (e) $(b^x)^p = b^{px}$ so $\log_b((b^x)^p) =$ _____. Reformulated: _____

   (f) We can also change the base of our logarithm! Add the final answer to the next question here: _____

3. What if we want to change the base of our logarithm from $b > 0$ to $c > 0$?

   (a) Let's start with $\log_b(x)$. Our goal is to get this equal to something with only logs of base c. To that end, how can we write x as a power of b? (You'll have a log in the exponent)

   (b) Use your answer to part (a) and property (e) of logs above to simplify $\log_c(x)$.

   (c) Solve for $\log_b(x)$.

4. Find the domain of $\log_5(5^{x+1})$, then simplify the expression.

5. Find the domain of $5^{\log_5(x+1)}$, then simplify the expression.

6. Find the domain of $\log_{1/2}(2^{9x})$, then simplify the expression.

7. Find the domain of $\log\left(\dfrac{x^3}{2\sqrt{y}}\right)$, then simplify the expression.

8. Simplify the expression $\log_2(10)$.

9. Simplify the expression $\log_6(45) + \log_6(28) - \log_6(35)$.

10. Simplify the expression $\ln(e^{-5})$.

11. Simplify the expression $\ln\left(\ln\left(e^{e^9}\right)\right)$.

12. Solve the equation $10^{2x+1} = 7$.

13. Solve the equation $\ln(x+1) + \ln(x+2) = \ln(6^x)$.